Long-periodic Precession Parameters

(my personal contribution to a prepared paper by N. Capitaine, P. Wallace & J. Vondrák)

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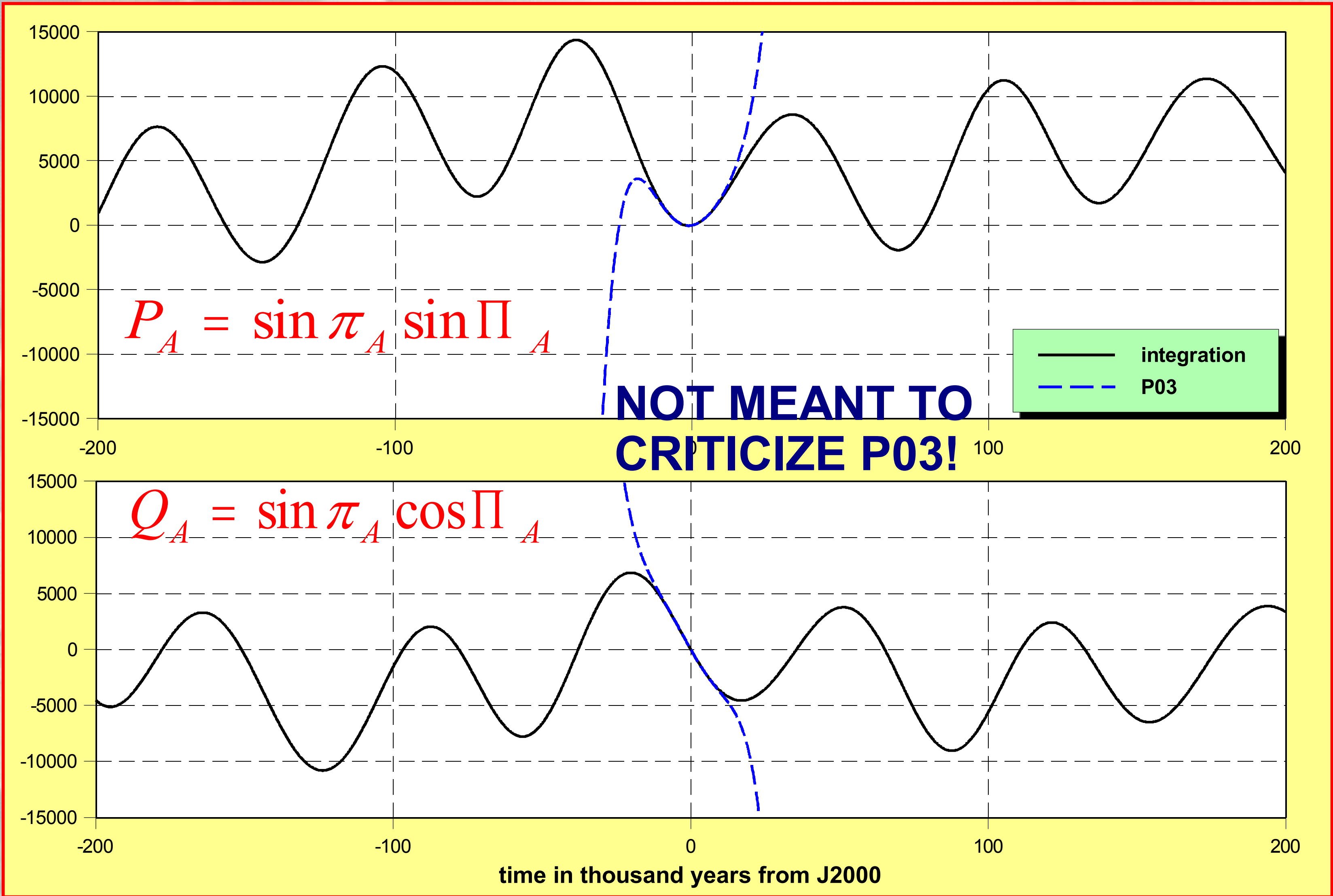
Outline:

- ♦ **Motivation;**
- ♦ **Long-periodic expressions for:**
  - ♦ **precession of the ecliptic;**
  - ♦ **general precession and obliquity;**
- ♦ **Conclusions.**

Motivation:

- All models of precession used so far, including the newest P03, are expressed as
  - polynomials.
- Therefore, the model P03 is very accurate, but only for a limited time period (several centuries around J2000):
  - the errors quickly grow with time!
- In reality, all precession parameters describe very long-periodic processes, with periods equal to hundreds of centuries:
  - It can be demonstrated, e.g., by numerical integration of the respective equations of motion.

Example: precession of the ecliptic ["]



The goal of the present study:

- To find relatively simple expressions that would
  - be very close or identical with those of P03 in a short-time scale ($\approx 10^3$ years);
  - follow approximately quasi-periodic variations of numerical integration in a long-term scale ($\approx 10^5$- 10^6 years).
- Such expressions could be used universally, for any instant in the scale of up to million years.
- This study is only an initial step to demonstrate how we can proceed in future, to achieve the final goal:
  - The next generation of precession model - long-term expressions for the CIP motion (*X, Y*).

A. Precession of the ecliptic:

Basis: Numerical integration of the motion of the solar system, using the integrator package Mercury 6 (Chambers, MNRAS 1999), in the interval ±2000cy with the step equal to 1cy.

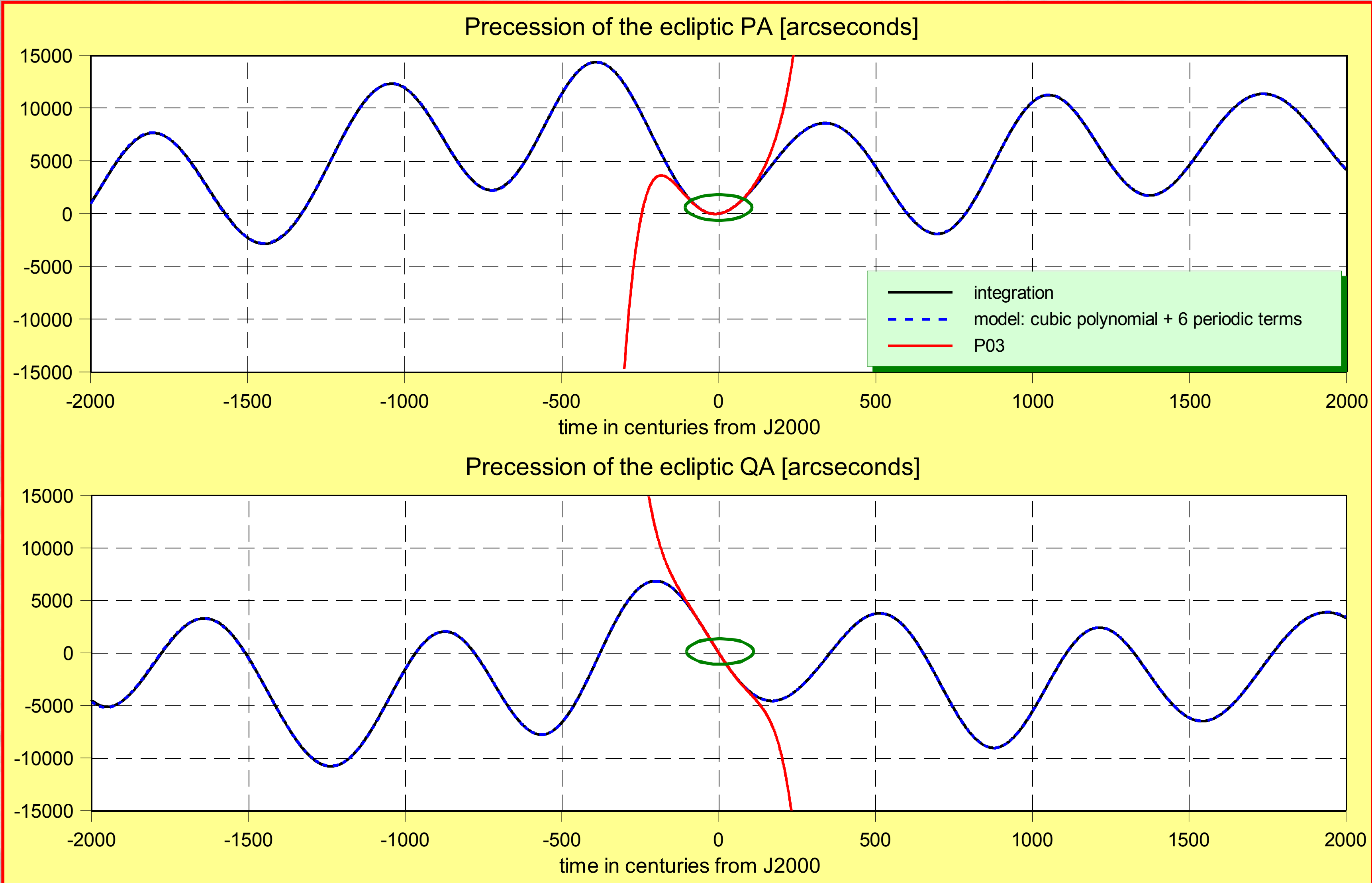
- **The model:**
  - **6 long-periodic terms, corresponding to Laskar's largest terms with periods 2309, 1831, 730, 708, 668 and 492 centuries;**
  - **+ cubic polynomial to account for longer periods.**
  - **The terms above are fitted to**
    - **numerical integration outside the interval ±10cy from J2000;**
    - **P03 values inside this interval, with higher weights;**
  - **Small additional corrections are applied to constant, linear and quadratic terms to keep derivatives up to 2nd order identical with P03.**

Long-term expressions for the precession of the ecliptic:

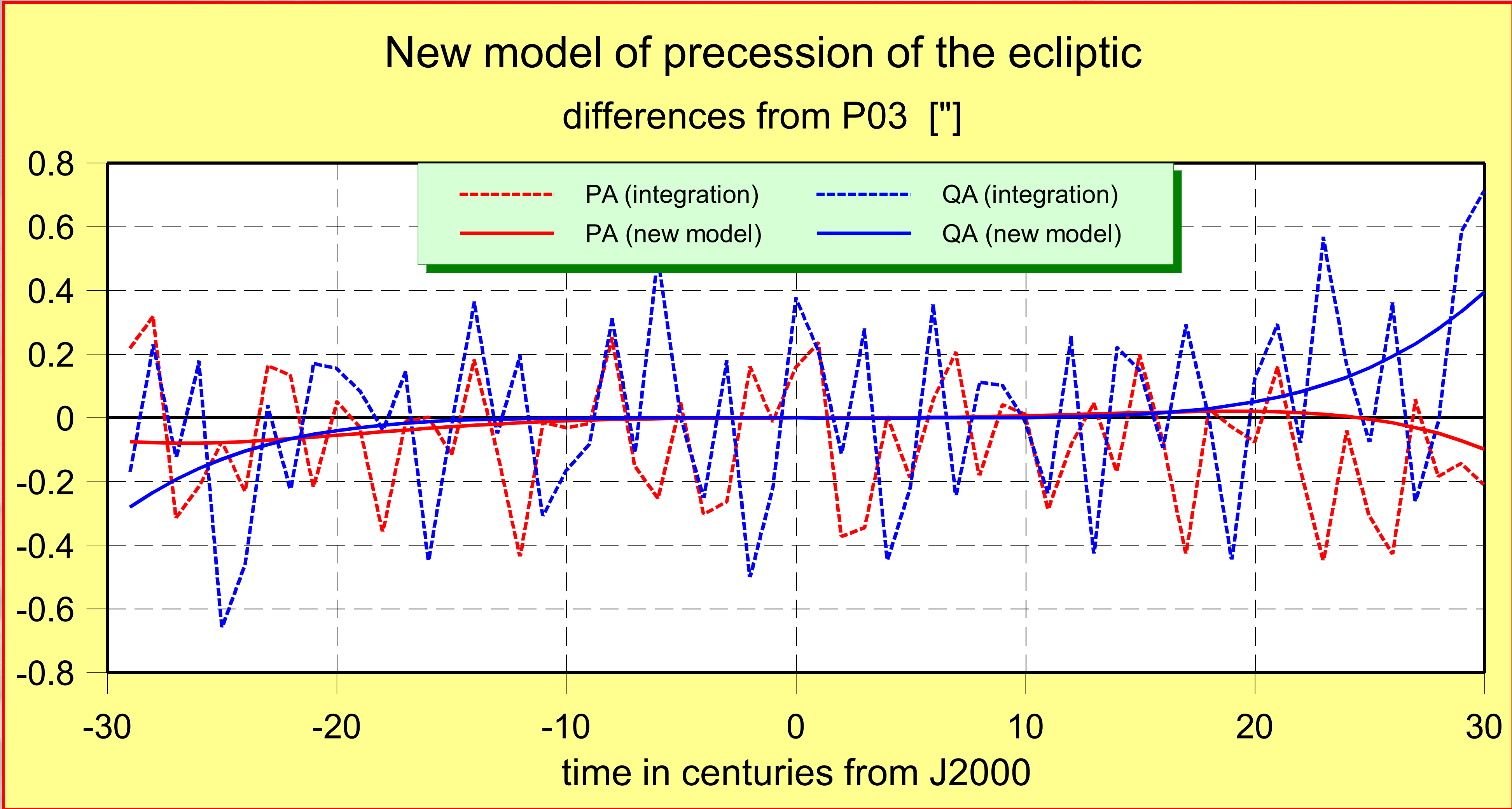
$$P_A = 5750.804069 + 0.1948311T - 0.00016739T^2 - 48 \times 10^{-9} T^3 + \sum_1^6 \left(C_i \cos 2\pi T / P_i + S_i \sin 2\pi T / P_i \right)$$

$$Q_A = -1673.999018 + 0.3474459T + 0.00011243T^2 - 64 \times 10^{-9} T^3 + \sum_1^6 \left(C_i \cos 2\pi T / P_i + S_i \sin 2\pi T / P_i \right)$$

term		P_A	Q_A	P [cy]
s_1	C_1	486.230527	2559.065245	2308.98
	S_1	-2578.462809	485.116645	
s_2	C_2	-963.825784	247.582718	1831.25
	S_2	-237.405076	-971.375498	
s_3	C_3	-1868.737098	-957.399054	687.52
	S_3	1007.593090	-1930.464338	
s_4	C_4	-1589.172175	493.021354	729.97
	S_4	-423.035168	-1634.905683	
s_6	C_5	429.442489	-328.301413	492.21
	S_5	337.266785	429.594383	
	C_6	-2244.742029	-339.969833	708.13
	S_6	221.240093	-2131.745072	


Precession of the ecliptic PA [arcseconds]
15000
10000
5000
0
-5000
-10000
-15000
-2000
-1500
-1000
-500
0
500
1000
1500
2000
time in centuries from J2000
integration
model: cubic polynomial + 6 periodic terms
P03
Precession of the ecliptic QA [arcseconds]
time in centuries from J2000

V C D Z
C E D R



New model of precession of the ecliptic
differences from P03 ["]
PA (integration)
QA (integration)
PA (new model)
QA (new model)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-30
-20
-10
0
10
20
30
time in centuries from J2000

B. General precession, obliquity

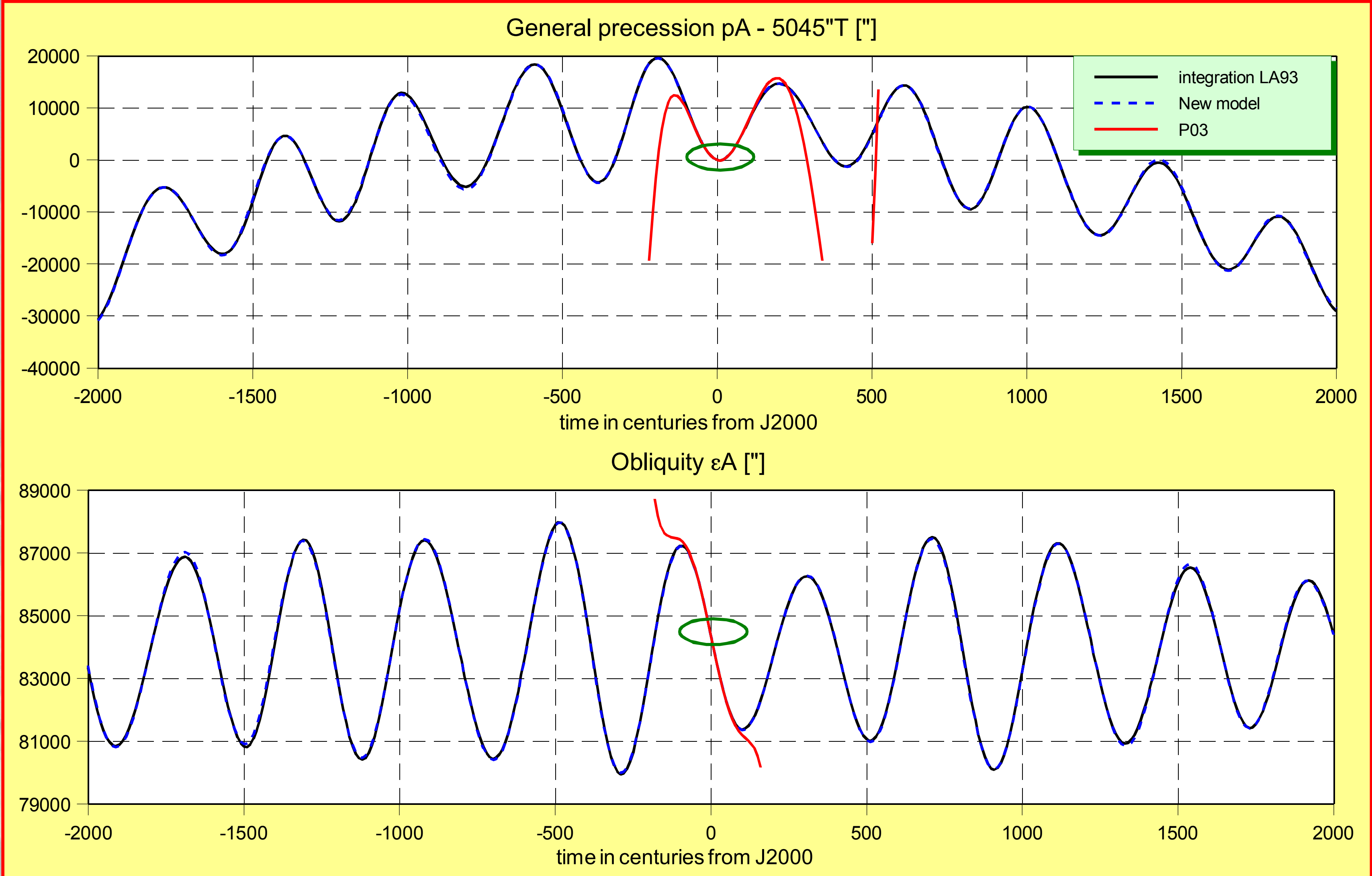
Basis: Numerical integration of general precession and obliquity LA93 (Laskar et al., A&A 1993) available in interval ±1My with the step equal to 10cy. Additional corrections applied to account for the change of dynamical ellipticity, J2 dot and secular change of obliquity.

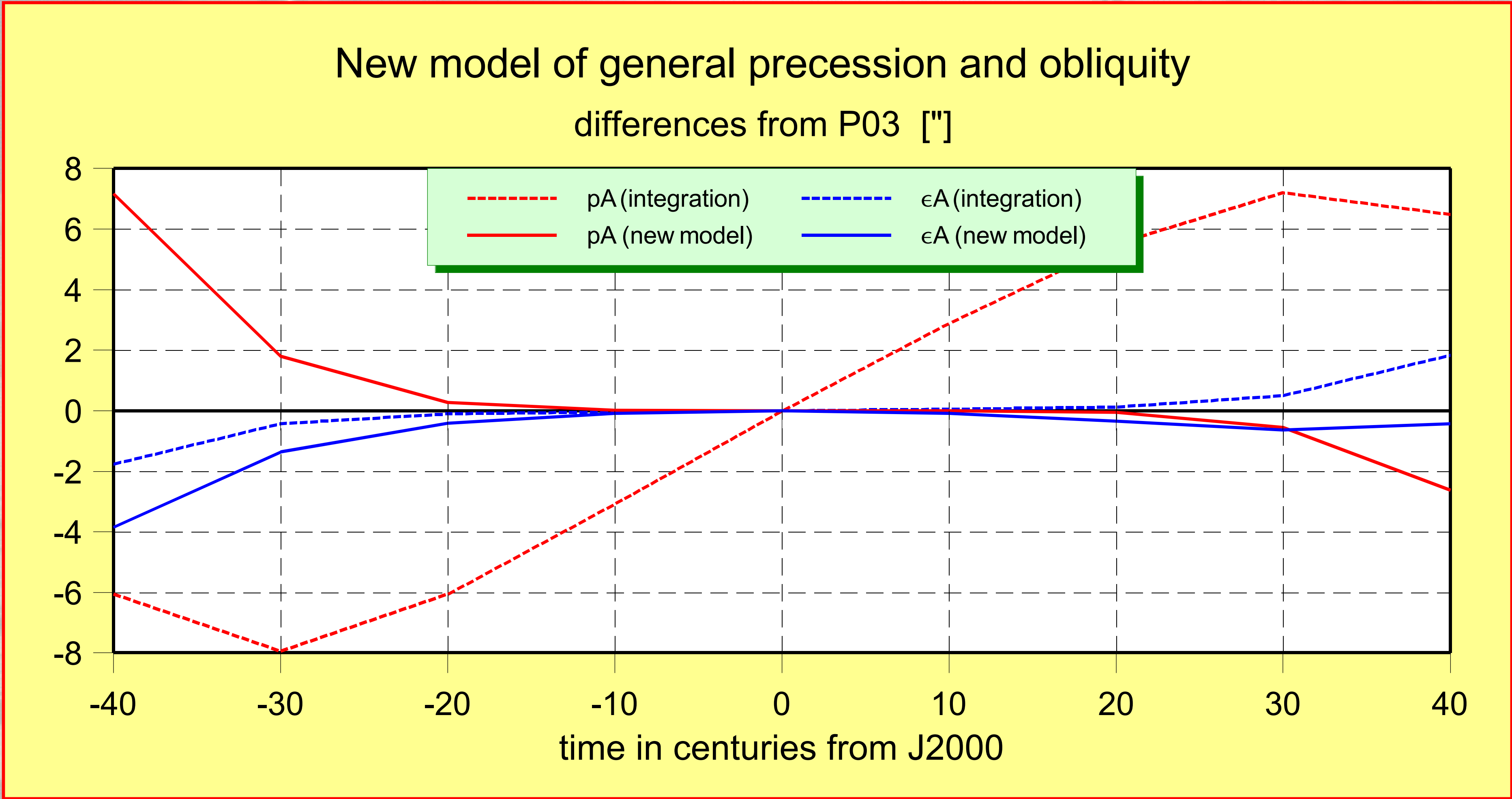
- ♦ **The model:**
  - ♦ **10 long-periodic terms, corresponding to periods equal to 4043, 537, 417, 410, 403, 396, 305, 289, 281 and 204 cy;**
  - ♦ **+ cubic polynomial to account for longer periods.**
    - ♦ **These terms are fitted to numerical integration outside the interval ±10cy from J2000 and P03 values inside this interval,**
    - ♦ **small additional corrections are applied to constant, linear and quadratic terms (the latter for precession only) to keep derivatives up to 2nd order identical with P03.**

Long-term expressions for the general precession & obliquity:

$$p_A = 7907.295950 + 5044.374034T - 0.00713473T^2 + 6\times10^{-9}T^3$$
$$+\sum_1^{10}\left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$
$$\varepsilon_A = 83973.876448 - 0.0425899T - 0.00000113T^2$$
$$+\sum_1^{10}\left(C_i \cos 2\pi T/P_i + S_i \sin 2\pi T/P_i\right)$$

term		p_A	ϵ_A	P [cy]
$p+s_3$	C_1	-6180.062400	807.904635	409.90
	S_1	-2434.845716	-2056.455197	
$p+s_4$	C_2	-2721.869299	-177.959383	396.15
	S_2	538.034071	-912.727303	
$p+s_6$	C_3	1460.746498	371.942696	536.91
	S_3	-1245.689351	447.710000	
$p+\sigma_3$	C_4	-1838.488899	-176.029134	402.90
	S_4	529.220775	-611.297411	
$p+\sigma_6$	C_5	949.518077	-89.154030	417.15
	S_5	277.195375	315.900626	
$p+s_1$	C_6	32.701460	-336.048179	288.92
	S_6	945.979710	12.390157	
g_2-g_5	C_7	598.054819	-17.415730	4042.97
	S_7	-955.163661	-15.922155	
	C_8	-293.145284	-28.084479	304.90
	S_8	93.894079	-102.870153	
	C_9	66.354942	21.456146	281.46
	S_9	0.671968	24.123484	
	C_{10}	18.894136	30.917011	204.38
	S_{10}	-184.663935	2.512708	


General precession pA - 5045"T ["]
20000
10000
0
-10000
-20000
-30000
-40000
-2000
-1500
-1000
-500
0
500
1000
1500
2000
time in centuries from J2000
integration LA93
New model
P03
Obliquity εA ["]
89000
87000
85000
83000
81000
79000
-2000
-1500
-1000
-500
0
500
1000
1500
2000
time in centuries from J2000


New model of general precession and obliquity
differences from P03 ["]
pA (integration)
pA (new model)
εA (integration)
εA (new model)
8
6
4
2
0
-2
-4
-6
-8
-40
-30
-20
-10
0
10
20
30
40
time in centuries from J2000

Conclusions:

- It is possible to construct a model of precession that:
  - is as accurate as P03 in short-time interval (a few centuries) around J2000,
  - approximately follows the periodical character of the phenomenon in a long-term sense (hundreds of thousand years).
- This work is meant as an introductory study for a prepared paper by Capitaine, Wallace and Vondrák devoted to this problem.

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